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Light-cone formulation of conformal field theory adapted to AdS/CFT correspondence

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Abstract

Light-cone formulation of conformal field theory in space-time of arbitrary dimension is developed. Conformal fundamental and shadow fields with arbitrary conformal dimension and arbitrary spin are studied. Representation of conformal algebra generators on space of conformal fundamental and shadow fields in terms of spin operators which enter in light-cone gauge formulation of field dynamics in AdS space is found. As an example of application of light-cone formalism we discuss AdS/CFT correspondence for massive arbitrary spin AdS fields and corresponding boundary CFT fields at the level of two point function.

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1 Introduction

Conjectured duality [1] of large N conformal $\mathcal{N}=4$ SYM theory and type IIB superstring theory in $AdS_5\times S^5$ has triggered intensive study of field (string) dynamics in AdS space and conformal field theory. By now it is clear that in order to understand the conjectured duality better it is necessary to develop powerful approaches to study of field (string) dynamics in AdS space as well as conformal field theory. Light-cone approach is one of the promising approaches which might be helpful to understand AdS/CFT duality better. As is well known, quantization of Green-Schwarz superstrings propagating in flat space is straightforward only in the light-cone gauge. Since, by analogy with flat space, we expect that a quantization of the Green-Schwarz AdS superstring with a Ramond - Ramond charge [2] will be straightforward only in the light-cone gauge [3], it seems that from the stringy perspective of AdS/CFT correspondence the light-cone approach to conformal field theory is the fruitful direction to go. Light-cone formulation of *totally symmetric conformal fundamental fields with canonical conformal dimension* and *shadow fields* was obtained in [4]. In this letter we develop light-cone formulation for *conformal fundamental and shadow fields with arbitrary conformal dimension and arbitrary type of symmetry* (totally symmetric and mixed symmetry).

Let us first formulate the main problem we solve in this letter. Conformal fundamental fields in $d-1$ dimensional space-time are associated with representations of $SO(d-1,2)$ group labelled by Δ, eigenvalue of the dilatation operator, and by $\mathbf{h}=(h_1,\ldots h_\nu)$, $\nu=[\frac{d-1}{2}]$, which is the highest weight of the unitary representation of the $SO(d-1)$ group. The highest weights h_i are integers and half-integers for bosonic and fermionic fileds respectively. The conformal dimension Δ and $\mathbf{h}$ satisfy the restriction

$$\Delta\geq\Delta_0\,,\tag{1.1}$$

where the canonical conformal dimension Δ_0 is given by[1]

$$\Delta_0=h_k-k-2+d\,,\tag{1.2}$$

and a number k is defined from the relation[2]

$$h_1=\ldots=h_k>h_{k+1}\geq h_{k+2}\geq\ldots\geq h_\nu\geq 0,\tag{1.3}$$

where for odd d the weight h_ν should be replaced by $|h_\nu|$ (see e.g.[6]). Another representations of the conformal group associated with so called shadow fields have eigenvalue of dilatation operator $\tilde\Delta$ given by

$$\tilde\Delta=d-1-\Delta\,.\tag{1.4}$$

Bosonic (fermionic) fields with $\mathbf{h}=(h_1,0,\ldots,0)$ ($\mathbf{h}=(h_1,1/2,\ldots,1/2,h_\nu)$, $h_\nu=\pm1/2$ for odd d, $h_\nu=1/2$ for even d) are referred to as totally symmetric conformal fundamental and shadow fields [3]. In manifestly Lorentz covariant formulation the bosonic(fermionic) totally symmetric conformal and shadow fields are described by a set of the tensor (tensor-spinor) fields whose

[1]Unitarity restriction (1.2) of the $so(d-1,2)$ algebra for the cases $d=4,5$ was studied in [5, 6] and for arbitrary d in [7]. For discussion of unitary representations of various superalgebras see e.g.[8, 9].

[2]The labels h_i are the standard Gelfand-Zeitlin labels. They are related with Dynkin labels h_i^D by formula: $(h_1^D,h_2^D,\ldots,h_{\nu-1}^D,h_\nu^D)=(h_1-h_2,h_2-h_3,\ldots,h_{\nu-1}-h_\nu,h_{\nu-1}+h_\nu)$.

[3]We note that $\Delta=d-2$, $\mathbf{h}=(1,0,\ldots,0)$ and $\Delta=d-1$, $\mathbf{h}=(2,0,\ldots,0)$ correspond to conserved vector current and conserved traceless spin two tensor field (energy-momentum tensor) respectively. Conserved conformal currents can be built from massless scalar, spinor and spin 1 fields (see e.g. [10]). The shadow fields with $\tilde\Delta=2-s$, $\mathbf{h}=(s,0,\ldots,0)$ can be used to formulate higher derivatives conformally invariant equations of motion for spin s totally symmetric tensor fields (see e.g. [11]-[14]). Discussion of conformally invariant equations for mixed symmetry tensor fields with discrete Δ may be found in [15].

$SO(d-2,1)$ space-time tensor indices have the structure of the respective Young tableauxes with one row. Lorentz covariant description of totally symmetric arbitrary spin conformal fundamental fields with $\Delta = \Delta_0$ and shadow fields with $\tilde{\Delta} = d-1-\Delta_0$ is well known. Light-cone description of such fields was developed in [4]. Bosonic (fermionic) fields with $h_2 > 0$ ($|h_2| > 1/2$) are referred to as mixed symmetry fields. In this paper we develop light-cone formulation of CFT [4] which is applicable to description of mixed symmetry conformal fundamental (and shadow) fields with arbitrary conformal dimension $\Delta > \Delta_0$ and for arbitrary space dimensions.

Remarkable feature of light-cone approach to CFT we exploit is that number of physical spin D.o.F. of massless field in d-dimensional space-time coincides with the number of independent spin degrees of freedom of the corresponding conformal fundamental field (and shadow field) with $\Delta = \Delta_0$ in $d-1$ -dimensional space-time[4]. The same coincidence holds true for massive field in d-dimensional space and the corresponding conformal fundamental field (and shadow field) in $d-1$-dimensional space with anomalous conformal dimension, $\Delta > \Delta_0$. It is this fact that allows us to develop the representation for generators of the conformal algebra $so(d-1,2)$ acting in space of conformal fundamental and shadow fields in terms of spin operators which enters in light-cone gauge description of fields propagating in AdS space time. This is a reason why we refer to such representation for CFT generators as AdS friendly representation.

2 Light-cone from of field dynamics in AdS space

In order to demonstrate explicit parallel between light-cone formulation of field dynamics in AdS space and that of conformal field theory we begin with discussion of light-cone formulation of field dynamics in AdS space developed in [4, 16]. Let $\phi(x)$ be a bosonic arbitrary spin field propagating in AdS_d space. If we collect spin degrees of freedom in a ket-vector $|\phi\rangle$ then a light-cone gauge action for ϕ can be cast into the following form[4][5]

$$S_{l.c.} = \frac{1}{2}\int d^dx \langle\phi|(\Box - \frac{1}{z^2}A)|\phi\rangle\,, \qquad \Box = 2\partial^+\partial^- + \partial^i\partial^i + \partial_z^2\,. \tag{2.1}$$

An operator A being independent of space-time coordinates and their derivatives is referred to as AdS mass operator. This operator acts only on spin indices of $|\phi\rangle$.

We turn now to discussion of global $so(d-1,2)$ symmetries of the light-cone gauge action. The choice of the light-cone gauge spoils the manifest global symmetries, and in order to demonstrate that these global invariances are still present one needs to find the Noether charges which generate them. Noether charges (or generators) can be split into kinematical and dynamical generators. In this paper we deal with free fields. At a quadratic level both kinematical and dynamical generators have the following standard representation in terms of the physical light-cone field

$$\hat{G} = \int dx^- d^{d-2}x \langle\partial^+\phi|G|\phi\rangle\,. \tag{2.2}$$

[4]For the case of totally symmetric tensor fields this simple fact can be checked in a rather straightforward way (see e.g.[4]). It is naturally to expect that light-cone matching of spin degrees of freedom is still to be case for the case of mixed symmetry AdS massless fields (which are beyond scope of this paper) and corresponding conformal fundamental and shadow fields.

[5]We use parametrization of AdS_d space in which $ds^2 = (-dx_0^2 + dx_i^2 + dx_{d-1}^2 + dz^2)/z^2$. Light-cone coordinates in $\pm$ directions are defined as $x^\pm = (x^{d-1} \pm x^0)/\sqrt{2}$ and x^+ is taken to be a light-cone time. We adopt the conventions: $\partial^i = \partial_i \equiv \partial/\partial x^i$, $\partial_z \equiv \partial/\partial z$, $\partial^\pm = \partial_\mp \equiv \partial/\partial x^\mp$, $z \equiv x^{d-2}$ and use indices $i,j = 1,\ldots,d-3$; $I,J = 1,\ldots,d-2$. Vectors of $so(d-2)$ algebra are decomposed as $X^I = (X^i, X^z)$.

Representation for the kinematical generators in terms of differential operators G acting on the physical field $|\phi\rangle$ is given by

$$P^i = \partial^i\,, \qquad P^+ = \partial^+\,, \tag{2.3}$$

$$D = x^+ P^- + x^-\partial^+ + x^I\partial^I + \frac{d-2}{2}\,, \tag{2.4}$$

$$J^{+-} = x^+ P^- - x^-\partial^+\,, \tag{2.5}$$

$$J^{+i} = x^+\partial^i - x^i\partial^+\,, \tag{2.6}$$

$$J^{ij} = x^i\partial^j - x^j\partial^i + M^{ij}\,, \tag{2.7}$$

$$K^+ = -\frac{1}{2}(2x^+x^- + x^J x^J)\partial^+ + x^+ D\,, \tag{2.8}$$

$$K^i = -\frac{1}{2}(2x^+x^- + x^J x^J)\partial^i + x^i D + M^{iJ}x^J + M^{i-}x^+\,, \tag{2.9}$$

while a representation for the dynamical generators takes the form

$$P^- = -\frac{\partial^I\partial^I}{2\partial^+} + \frac{1}{2z^2\partial^+}A\,, \tag{2.10}$$

$$J^{-i} = x^-\partial^i - x^i P^- + M^{-i}\,, \tag{2.11}$$

$$K^- = -\frac{1}{2}(2x^+x^- + x_I^2)P^- + x^- D + \frac{1}{\partial^+}x^I\partial^J M^{IJ} - \frac{x^i}{2z\partial^+}[M^{zi}, A] + \frac{1}{\partial^+}B\,, \tag{2.12}$$

where $M^{-i} = -M^{i-}$ and

$$M^{-i} \equiv M^{iJ}\frac{\partial^J}{\partial^+} - \frac{1}{2z\partial^+}[M^{zi}, A]\,. \tag{2.13}$$

Operators A, B, M^{IJ} are acting only on spin degrees of freedom of the field $|\phi\rangle$. $M^{IJ} = M^{ij}, M^{zi}$ are spin operators of the $so(d-2)$ algebra

$$[M^{IJ}, M^{KL}] = \delta^{JK}M^{IL} + 3\text{ terms}\,, \qquad M^{IJ\dagger} = -M^{IJ}\,, \tag{2.14}$$

while the operators A and B admit the following representation

$$A \;=\; 2B^z + 2M^{zi}M^{zi} + \frac{1}{2}M^{ij}M^{ij} + \langle C_{AdS}\rangle + \frac{d(d-2)}{4}\,, \tag{2.15}$$

$$B \;=\; B^z + M^{zi}M^{zi}\,. \tag{2.16}$$

$\langle C_{AdS}\rangle$ is eigenvalue of the second order Casimir operator of the $so(d-1,2)$ algebra for the representation labelled by $D(E_0, \mathbf{h})$:

$$\langle C_{AdS}\rangle = E_0(E_0 + 1 - d) + \sum_{\sigma=1}^{\nu} h_\sigma(h_\sigma - 2\sigma + d - 1)\,, \tag{2.17}$$

while B^z is z-component of $so(d-2)$ algebra vector B^I which satisfies the defining equation[6]

$$[B^I, B^J] = \Big(\langle C_{AdS}\rangle + \frac{1}{2}M^2 + \frac{d^2 - 5d + 8}{2}\Big)M^{IJ} - (M^3)^{[I|J]}\,. \tag{2.18}$$

[6] We use the notation $(M^3)^{[I|J]} \equiv \frac{1}{2}M^{IK}M^{KL}M^{LJ} - (I \leftrightarrow J)$, $M^2 \equiv M^{IJ}M^{IJ}$.

Equation (2.18) is a basic equation of light-cone gauge form of the relativistic dynamics in AdS space. General method of solving this equation may be found in Ref.[17]. As was noted the operator B^I transforms in vector representation of the $so(d-2)$ algebra

$$[B^I, M^{JK}] = \delta^{IJ} B^K - \delta^{IK} B^J\,. \tag{2.19}$$

One can check then that the light-cone gauge action (2.1) is invariant with respect to the global symmetries generated by $so(d-1,2)$ algebra taken to be in the form $\delta_{\hat{G}}|\phi\rangle = G|\phi\rangle$.

As seen AdS mass operator plays important role in light-cone approach. This operator is fixed by equations (2.15),(2.18). In study of AdS/CFT correspondence it is desirable to know explicit diagonalized operator A taken to be in the form

$$A = \kappa^2 - \frac{1}{4}\,. \tag{2.20}$$

The explicit diagonalized form of the operator κ is known for the following cases:
i) arbitrary spin totally symmetric and antisymmetric massless fields in AdS_d [4];
ii) type IIB supergravity in $AdS_5 \times S^5$ and $AdS_3 \times S^3$ backgrounds [18, 19];
iii) mixed symmetry arbitrary spin massless and self-dual massive fields in AdS_5 [20];

We discuss now operator κ for mixed symmetry arbitrary spin massive field in AdS_d. Massive field $|\phi\rangle$ associated with representation $D(E_0, \mathbf{h})$ transforms in irreps of $so(d-1)$ algebra where $\mathbf{h}$ is the highest weight of the unitary representation of the $so(d-1)$ algebra. This field can be decomposed into representation of $so(d-3) \times so(2)$ subalgebra as follows (see formula (5.38) in [4])

$$|\phi\rangle = \sum_{s'} \oplus |\phi_{s'}\rangle \qquad s' = \left\{ \begin{array}{ll} 0\,, \pm 1, \ldots, \pm h_k\,, & \hbox{for bosonic fields,} \\ \pm\frac{1}{2}, \ldots, \pm h_k\,, & \hbox{for fermionic fields,} \end{array} \right. \tag{2.21}$$

where $|\phi_{s'}\rangle$ are representations[7] of the $so(d-3)$ algebra and s' is eigenvalue of the $so(2)$ algebra generator. On the whole space of $|\phi_{s'}\rangle$ the operator κ takes eigenvalue

$$\kappa_{s'} = E_0 - \frac{d-1}{2} + s'\,, \tag{2.22}$$

where lowest energy value E_0 is expressible in terms of standard mass parameter as [16]:

$$E_0 = \frac{d-1}{2} + \sqrt{m^2 + \Big(h_k - k + \frac{d-3}{2}\Big)^2}\,, \qquad \hbox{for bosonic fields;} \tag{2.23}$$

$$E_0 = m + h_k - k - 2 + d\,, \qquad \hbox{for fermionic fields}\,, \tag{2.24}$$

and the number k is defined from the relation (1.3).

3 Light-cone form of conformal field theory

We present now light-cone formulation of conformal field theory. In Ref.[4] we have developed light-cone formulation of totally symmetric conformal fundamental fields with canonical conformal dimension Δ_0 and associated shadow fields starting with Lorentz covariant formulation. This

[7] For the case of totally symmetric massive bosonic (fermionic) fields the $|\phi_{s'}\rangle$ is irreducible spin $|s'|$ ($|s'| - \frac{1}{2}$) tensor (tensor-spinor) of the $so(d-3)$ algebra. For mixed symmetry fields the $|\phi_{s'}\rangle$ is reducible representation of the $so(d-3)$ algebra in general.

strategy is difficult to realize in many cases because the Lorentz covariant formulations are not available in general. One of attractive features of light-cone formalism is that it admits to formulate CFT without knowledge of Lorentz covariant formulation. The practice we have got while deriving light-cone formulation of totally symmetric conformal fundamental and shadow fields allows us to develop general light-cone formalism. In this section we construct light-cone form of the $so(d-1,2)$ conformal algebra generators for arbitrary conformal dimension Δ (1.1) and arbitrary symmetry (totally symmetric and mixed symmetry) conformal fundamental and shadow fields. We show that these generators can be constructed in terms of the spin operators and the AdS mass operator which appear in light-cone gauge formulation of field dynamics in AdS space. We start with discussion of bosonic fields.

To develop general light-cone we should make an assumption about generators. Based on our previous study of totally symmetric conformal fundamental and shadow fields (see Appendix C of Ref.[4]) we make the following two assumptions about structure of the conformal algebra generators acting on conformal fundamental and shadow fields[8] .

(i) Taking into account the form of the generators P^a, J^{ab}, D found for the case of totally symmetric representations we suppose that they maintain this form for arbitrary representations

$$P^a = \partial^a \,, \tag{3.1}$$

$$J^{+i} = l^{+i} \,, \tag{3.2}$$

$$J^{+-} = l^{+-} \,, \tag{3.3}$$

$$J^{ij} = l^{ij} + M^{ij} \,, \tag{3.4}$$

$$J^{-i} = l^{-i} + M^{-i} \,, \tag{3.5}$$

$$D = x^a\partial_a + \frac{d-2}{2} \,, \tag{3.6}$$

where we use the notation[9]

$$l^{ab} \equiv x^a\partial^b - x^b\partial^a \,, \tag{3.7}$$

$$K_0^a \equiv -\frac{1}{2}x^2\partial^a + x^a x^b\partial_b \,, \tag{3.8}$$

$$M^{-i} \equiv M^{ij}\frac{\partial^j}{\partial^+} + \frac{q}{\partial^+}M^i \,, \tag{3.9}$$

$$q \equiv \sqrt{\partial_a\partial^a} \,, \qquad x^2 \equiv x_a x^a \,. \tag{3.10}$$

M^{ij} and M^i are the spin operators, i.e. they are acting only on the spin degrees of freedom of CFT fields. The operators M^{ij} satisfy commutators of the $so(d-3)$ algebra, while the operators M^{ij} and M^i satisfy commutators of the $so(d-2)$ algebra:

$$[M^{ij}, M^{kl}] = \delta^{jk}M^{il} + 3 \text{ terms} \,, \qquad [M^i, M^j] = M^{ij} \,,$$
$$[M^i, M^{jk}] = \delta^{ij}M^k - \delta^{ik}M^j \,. \tag{3.11}$$

[8]Commutation relations of the $so(d-1,2)$ algebra we use may be found in Eqs.(2.3)-(2.6) of Ref.[4].

[9]Lorentz basis coordinates of $d-1$ dimensional flat space x^a, $a = 0,1,\ldots,d-3,d-1$ are decomposed in light-cone basis coordinates $x^\pm$, x^i, $i = 1,\ldots d-3$, $x^\pm \equiv (x^{d-1} \pm x^0)/\sqrt{2}$. The derivatives $\partial_a \equiv \partial_{x^a}$ are decomposed as $\partial_+ = \partial_{x^+} = \partial^-$, $\partial_- = \partial_{x^-} = \partial^+$ and $\partial_i = \partial_{x^i} = \partial^i$.

These operators subject the following hermitian conjugation rules

$$M^{ij\dagger} = -M^{ij}\,, \qquad M^{i\dagger} = M^i\,. \tag{3.12}$$

(ii) The generator K^+ has the following form

$$K^+ = K_0^+ + \frac{d-2}{2}x^+ - \frac{\partial^+}{2q^2}A\,. \tag{3.13}$$

Note that we do not make assumptions about the form of the remaining generators K^i and K^-. Now the problem we solve is formulated as follows. Given the spin operators M^{ij}, M^i find the operator A (3.13) and the remaining generators K^i, K^-. In Appendix A by exploiting only the commutation relations of the conformal algebra $so(d-1,2)$ we demonstrate that the above assumptions turn out to be sufficient to find the remaining generators and to get closed defining equations for the operator A. We present our final result for the generators K^i and K^-

$$K^i = K_0^i + \frac{d-2}{2}x^i - \frac{\partial^i}{2q^2}A + M^{ij}x^j - x^+M^{-i} + \frac{1}{2q}[M^i, A]\,, \tag{3.14}$$

$$\begin{aligned} K^- &= K_0^- + \frac{d-2}{2}x^- - \frac{\partial^-}{2q^2}A + \frac{1}{\partial^+}(M^{ij}x^i\partial^j + M^ix^iq) \\ &- \frac{\partial^i}{2q\partial^+}[M^i, A] + \frac{1}{\partial^+}B\,, \end{aligned} \tag{3.15}$$

where we introduce new spin operator B. By definition this operator acts only on the spin degrees of freedom of conformal fundamental and shadow fields. As in the case of the light-cone gauge formulation of fields dynamics in AdS space the operators A and B admit the representation

$$A = 2B^{0'} - 2M^iM^i + \frac{1}{2}M^{ij}M^{ij} + \langle C_{CFT}\rangle + \frac{d(d-2)}{4}\,, \tag{3.16}$$

$$B = B^{0'} - M^iM^i\,, \tag{3.17}$$

where vector B^I is decomposed as $B^I = (B^{0'}, B^i)$ and satisfies the basic equation[10]

$$-[B^I, B^J] = \Big(\langle C_{CFT}\rangle + \frac{1}{2}M^2 + \frac{d^2-5d+8}{2}\Big)M^{IJ} - (M^3)^{[I|J]}\,. \tag{3.18}$$

$\langle C_{CFT}\rangle$ is eigenvalue of the second order Casimir operator of the $so(d-1,2)$ algebra for the representation labelled by Δ and $\mathbf{h}$. The $\langle C_{CFT}\rangle$ is obtainable from (2.17) where we should make the replacement $E_0 \to \Delta$.

Comparing Eqs.(2.15)-(2.18) and (3.16)-(3.18) we conclude that the operators A and B in light-cone form of CFT satisfy the same equations as in light-cone gauge formulation of field dynamics in AdS space[11]. We note also that on CFT and AdS sides these operators are realized on fields having the same number of spin D.o.F. Because of the representation for CFT generators

[10] In this section the 'transverse' indices $I = \{0', i\}$, $J = \{0', j\}$ are contracted by using flat metric η^{IJ}: $\eta^{0'0'} = -1$, $\eta^{ij} = \delta^{ij}$. This is to say that expression $M^2 = M^{IJ}M^{IJ}$ takes the form $M^2 = M^{ij}M^{ij} - 2M^iM^i$, i.e. we use the identification $M^{0'i} \equiv M^i$.

[11] Some differences in signs in Eqs.(2.15)-(2.18) and Eqs.(3.16)-(3.18) are related with the fact we use anti-hermitian spin operator M^{zi} (2.14) on AdS side and hermitian M^i (3.12) on CFT side.

given in (3.1)-(3.15) is formulated in terms of the spin operators A, B, M^{IJ} which appear in light-cone gauge formulation of field dynamics in AdS space we refer to this representation for CFT generators as AdS friendly form of CFT[12].

Let us denote CFT field on which AdS friendly representation is realized as $|\mathcal{O}_{AdS}\rangle$. Generators acting on $|\mathcal{O}_{AdS}\rangle$ (see (3.5),(3.13)-(3.15)) are non-polynomial with respect to the derivatives in transverse and minus light-cone directions, ∂^i, ∂^-[13]. However, these non-polynomial terms can be cancelled by passing from basis of $|\mathcal{O}_{AdS}\rangle$ to the bases of conformal fundamental fields and shadow fields which we shall denote by $|\mathcal{O}\rangle$ and $|\widetilde{\mathcal{O}}\rangle$ respectively. We demonstrate cancellation of non-polynomial terms in bases of $|\mathcal{O}\rangle$ and $|\widetilde{\mathcal{O}}\rangle$ for the simplest case of generator K^+. To this end we note that the AdS mass operator A being hermitian can be diagonalized on the whole space of $|\mathcal{O}_{AdS}\rangle$. Being diagonalized this operator can be presented in the form given in (2.20). Now we make the following transformation to the bases of conformal fundamental and shadow fields

$$|\mathcal{O}_{AdS}\rangle = q^{-\omega}|\mathcal{O}\rangle\,, \qquad |\mathcal{O}_{AdS}\rangle = q^{-\omega}|\widetilde{\mathcal{O}}\rangle\,, \tag{3.19}$$

where we use the notation

$$\omega = \left\{ \begin{array}{lll} \kappa + \frac{1}{2} & \text{for conformal fundamental field} & |\mathcal{O}\rangle\,, \\ -\kappa + \frac{1}{2} & \text{for shadow field} & |\widetilde{\mathcal{O}}\rangle\,. \end{array} \right. \tag{3.20}$$

It easy to check that in the bases of conformal fundamental and shadow fields $|\mathcal{O}\rangle$, $|\widetilde{\mathcal{O}}\rangle$ the generator K^+ given in (3.13) takes the form

$$K^+ = K_0^+ + (\omega + \frac{d-2}{2})x^+\,, \tag{3.21}$$

where the respective values of ω are given in (3.20). Thus non-polynomial term $(1/q^2)A$ of expression K^+ (3.13) cancels out in bases of $|\mathcal{O}\rangle$ and $|\widetilde{\mathcal{O}}\rangle$, as it should be. We note also that in bases of $|\mathcal{O}\rangle$ and $|\widetilde{\mathcal{O}}\rangle$ the dilatation operator is given by

$$D = x^a\partial_a + \omega + \frac{d-2}{2}\,, \tag{3.22}$$

and the invariant scalar product on the space of $|\mathcal{O}\rangle$ and $|\widetilde{\mathcal{O}}\rangle$ takes the standard form

$$(\widetilde{\mathcal{O}},\mathcal{O}) = \int d^{d-1}x\langle\widetilde{\mathcal{O}}||\mathcal{O}\rangle\,. \tag{3.23}$$

4 AdS/CFT correspondence

After we have derived the light-cone formulation for both the bulk massive arbitrary spin AdS fields and the boundary CFT fields we are ready to demonstrate explicitly AdS/CFT correspondence. Euclidean version of this correspondence for various particular cases has been studied in [21]–[30]. Intertwining operator realization of AdS/CFT correspondence was investigated in [31]. For review and complete list of references see [32]. In this Section we study AdS/CFT correspondence for both the Lorentzian and Euclidean signatures.

[12]The relations (2.3)-(2.12) and (3.1)-(3.15) are applicable to the bosonic fields. Extension to the fermionic fields is to make replacement $x^- \to x^- + \frac{1}{2\partial^+}$ in these relations

[13]Non-polynomial terms wit respect to light-cone derivative ∂^+ are unavoidable in light-cone formulation.

Lorentzian signature. We begin with study of correspondence for AdS space of Lorentzian signature. Discussion of this correspondence for the scalar field may be found in [33] and for totally symmetric arbitrary spin massless fields in [4].

We demonstrate that boundary value of normalizable solution of bulk equations of motion is related to the conformal fundamental field $|\mathcal{O}\rangle$, while that of non-normalizable solution is related to the shadow field $|\widetilde{\mathcal{O}}\rangle$ (see [33, 4]). To this end we consider light-cone equations of motion which take the form

$$\Big(-\partial_z^2 + \frac{1}{z^2}(\kappa^2 - \frac{1}{4})\Big)|\phi\rangle = q^2|\phi\rangle\,, \tag{4.1}$$

and obtain the following normalizable and non-normalizable solutions

$$|\phi_{norm}(x,z)\rangle = Z_\kappa(qz)|\mathcal{O}_{AdS}(x)\rangle\,, \tag{4.2}$$

$$|\phi_{non-norm}(x,z)\rangle = Z_{-\kappa}(qz)|\mathcal{O}_{AdS}(x)\rangle\,, \tag{4.3}$$

where we use the notation $Z_\kappa(z) \equiv \sqrt{z}J_\kappa(z)$ and J_κ is Bessel function[14]. In (4.2) we use the notation for the boundary field $|\mathcal{O}_{AdS}\rangle$ since we are going to demonstrate that this field is indeed a carrier of AdS friendly representation discussed above. This is to say that AdS transformations for bulk field $|\phi\rangle$ lead to conformal theory transformations for boundary field $|\mathcal{O}_{AdS}\rangle$

$$G_{{}_{AdS}}|\phi_{norm}(x,z)\rangle = Z_\kappa(qz)G_{{}_{CFT}}|\mathcal{O}_{AdS}(x)\rangle\,. \tag{4.4}$$

Here and below we use the notation $G_{{}_{AdS}}$ and $G_{{}_{CFT}}$ to indicate the realization of the $so(d-1,2)$ algebra generators on the bulk field $|\phi\rangle$ (2.3)-(2.12) and conformal theory field $|\mathcal{O}_{AdS}\rangle$ (3.1)-(3.15) respectively. Let us demonstrate the matching (4.4) for the generator K^+. To this end we use the following general relation

$$\begin{aligned}(K_0^a - \frac{z^2}{2}\partial^a + x^a(y+z\partial_z))Z_\kappa(qz) \;&=\; Z_\kappa(qz)\Big(K_0^a - \frac{\partial^a}{2q^2}(\kappa^2 - \frac{1}{4}) + x^a(y+z\partial_z)\Big)\\ &+\; \frac{\partial^a}{q}(\partial_q Z_\kappa(qz))(\frac{d-2}{2} - y - z\partial_z)\end{aligned} \tag{4.5}$$

where K_0^a is given in (3.8). Adopting relation (4.5) for $y=\frac{d-2}{2}$, $a=+$, and using expressions for $K^+_{{}_{AdS}}$ (2.8), $K^+_{{}_{CFT}}$ (3.13), and taking into account that $|\mathcal{O}_{AdS}\rangle$ in (4.2) does not depend z we get immediately that $K^+_{{}_{AdS}}$ and $K^+_{{}_{CFT}}$ satisfy the relation (4.4)[15].

Taking into account relations (3.19) and (4.2),(4.3) we make sure that for small z one has the local interrelations

$$|\phi_{norm}(x,z)\rangle \sim z^{\kappa+\frac{1}{2}}|\mathcal{O}(x)\rangle\,, \qquad |\phi_{non-norm}(x,z)\rangle \sim z^{-\kappa+\frac{1}{2}}|\widetilde{\mathcal{O}}(x)\rangle\,. \tag{4.6}$$

i.e. it is the conformal fundamental field $|\mathcal{O}\rangle$ (and shadow field $|\widetilde{\mathcal{O}}\rangle$) that is the boundary value of $|\phi_{norm}\rangle$ (and $|\phi_{non-norm}\rangle$) as it should be. Relations (4.4),(4.6) explain desired AdS/CFT correspondence in Lorentzian signature.

[14] To keep discussion from becoming unwieldy here we restrict our attention to non-integer κ. In this case the solutions given in (4.2),(4.3) are independent.

[15] For the case of the generators P^a, J^{+i}, J^{+-}, J^{ij}, D matching (4.4) is obvious. This matching for the generators J^{-i}, K^i, K^- can be proved following procedure explored in Section 7 of Ref.[4].

Euclidean signature. Now we demonstrate AdS/CFT correspondence in the Euclidean signature at the level of two point function[16]. In light-cone gauge the arbitrary spin massive and massless fields in AdS_d are described by respective $so(d-1)$ and $so(d-2)$ tensor fields. The Euclidean light-cone gauge action[17] takes then the form

$$S_{l.c.}^E = \frac{1}{2}\int d^dx \Big(\langle d\phi||d\phi\rangle + \frac{1}{z^2}\langle\phi|A|\phi\rangle \Big)\,. \tag{4.7}$$

Attractive feature of this action is that there are no contractions of tensor indices of fields with those of space derivatives, i.e., the action looks like a sum of actions for 'scalar' fields with different mass terms. This allows us to extend the analysis of Ref.[34] in a rather straightforward way. Using Green's function method and the AdS mass operator given in (2.20) a solution to equations of motion

$$\Big(-\partial_{\mathbf{x}}^2 - \partial_z^2 + \frac{1}{z^2}(\kappa^2 - \frac{1}{4})\Big)|\phi(\mathbf{x},z)\rangle = 0\,, \qquad \mathbf{x} \equiv (x^1,\ldots,x^{d-1}) \tag{4.8}$$

is found to be

$$|\phi(\mathbf{x},z)\rangle = \int d\mathbf{x}' \frac{z^{\kappa+\frac{1}{2}}}{(z^2 + |\mathbf{x}-\mathbf{x}'|^2)^{\kappa+\frac{d-1}{2}}} |\widetilde{\mathcal{O}}(\mathbf{x}')\rangle\,. \tag{4.9}$$

As was expected (see (4.3),(4.6)) this solution behaves for $z\rightarrow 0$ like $z^{-\kappa+\frac{1}{2}}|\tilde{\mathcal{O}}(\mathbf{x})\rangle$. Plugging this solution into the action (4.7) and evaluating a surface integral gives

$$S_{l.c.}^E = \frac{1}{2}\int d\mathbf{x}d\mathbf{x}'\, \langle\tilde{\mathcal{O}}(\mathbf{x})|\frac{\kappa+\frac{1}{2}}{|\mathbf{x}-\mathbf{x}'|^{2\kappa+d-1}}|\tilde{\mathcal{O}}(\mathbf{x}')\rangle\,. \tag{4.10}$$

This is light-cone representation for two point function of conserved current. The $so(d-3)$ decomposition of (4.10) can be obtained by using (2.21),(2.22).

Conclusions. The results presented here should have a number of interesting applications and generalizations, some of which are:

i) In this paper we develop light-cone formulation for conformal fundamental fields and shadow fields. As is well known shadow fields can be used to formulate conformal invariant equations of motion. It would be interesting to apply light-cone approach to study of conformal invariant equations of motion for mixed symmetry conformal fields.

ii) The light-cone formulation we develop in this paper allows us to study of bosonic and fermionic conformal fundamental (and shadow) fields on an equal footing. It would be interesting to apply our approach to study of supersymmetric multiplets of superconformal algebra $psu(2,2|4)$ which are relevant in the study of AdS/CFT correspondence.

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[16]Discussion of AdS/CFT correspondence for spin one Maxwell field, $s=1$, and graviton, $s=2$, may be found in [34] and [22, 35] respectively. Euclidean version of the AdS/CFT correspondence for arbitrary spin massless fields was studied in [20] (see also [36]).

[17]Note that only in the remainder of this paper we use the Euclidean signature. For $d=4$ the AdS mass operator is equal to zero for all massless fields. Here we restrict our attention to the dimensions $d\geq 5$.

Appendix A Derivation of generators K^i, K^-.

In this appendix we outline procedure of derivation of representation for generators K^i, K^- given in (3.14),(3.15) and prove that the operators A, B and the spin operators M^{ij}, M^i should satisfy the following equations

$$[A, M^{ij}] = 0\,, \tag{A.1}$$

$$2\{M^i, A\} - [[M^i, A], A] = 0\,, \tag{A.2}$$

$$[M^i, [M^j, A]] - \{M^{il}, M^{lj}\} - \{M^i, M^j\} = 2\delta^{ij} B\,. \tag{A.3}$$

We proceed in the following way.

i) We note that the commutation relations of K^+ with the generators given in (3.1)-(3.6) imply that the operator A is independent of space-time coordinates x^a and their derivatives ∂^a and commutes with spin operator M^{ij}, i.e. we obtain (A.1).

ii) From commutator

$$[K^+, J^{-i}] = K^i \tag{A.4}$$

we find representation for K^i given in (3.14).

iii) Using expressions for K^+ (3.13) and K^i (3.14) we evaluate the commutator

$$[K^+, K^i] = \frac{\partial^+}{4q^3}(-2\{M^i, A\} + [[M^i, A], A])\,. \tag{A.5}$$

From this relation and the commutator $[K^+, K^i] = 0$ we obtain the constraint (A.2).

iv) Making use of (3.5) and (3.14) we evaluate then the commutator

$$\begin{aligned} &[J^{-i}, K^j] \\ &= \delta^{ij}\Big(K_0^- + \frac{d-2}{2}x^- - \frac{\partial^-}{2q^2}A + \frac{1}{\partial^+}(M^{kl}x^k\partial^l + M^l x^l q) - \frac{\partial^l}{2q\partial^+}[M^l, A]\Big) \\ &+ \frac{1}{2\partial^+}\Big([M^i, [M^j, A]] - \{M^{il}, M^{lj}\} - \{M^i, M^j\}\Big)\,. \end{aligned} \tag{A.6}$$

From this and the $so(d-1,2)$ algebra commutator $[J^{-i}, K^j] = \delta^{ij}K^-$ we find the representation for the generator K^- given in (3.15) provided the spin operators M^{ij}, the operators A and B satisfy the constraint given in (A.3). Note that this constraint gives definition of operator B in terms of basic operators which are spin operator M^{ij} and AdS mass operator A.

Thus we obtain representation for generators K^i, K^- given in (3.14),(3.15) and equations (A.1)-(A.3). All that remains is to prove that equations (A.1)-(A.3) are equivalent to representation for operators A and B and basic defining equations given in (3.16)-(3.18). This can be done by following a procedure explored in the Appendix A of Ref.[16].

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